

Quantum associative memory with improved distributed queries

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Abstract The paper proposes an improved quantum associative algorithm with distributed query based on model proposed by Ezhov *et al.*. We introduce two modifications of the query that optimized data retrieval of correct multi-patterns simultaneously for any rate of the number of the recognition pattern on the total patterns. Simulation results are given.

Keywords Hopfield model · quantum associative memory · pattern recognition · Grover's algorithm · distributed queries

1 Introduction

Since the last two decades, there is a growing interest in quantum computing, due to the improvement in memory size (such as super dense coding) and the speed-up in computing time. Three advantages of quantum computing make it possible: (i) the *quantum parallelism* which is expressed in the principle of superposition and provides an advantage in processing huge data sets; (ii) the *entanglement*, the strange quantum phenomenon that links qubits across distances and provides the possibility of measuring the state of all qubits in a register whose values are interdependent; (iii) the *unitary* of quantum gates which ensure reversibility and therefore overcome energy dissipation.

Associative memory is an important tool for intelligent control, artificial intelligence and pattern recognition. Considering that quantum information is information stored as a property of a quantum system e.g., the polarization of

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a photon, or the spin of an electron, several approaches of quantum associative memory for pattern recognition have been proposed.

Perus *et al.* worked with quantum-wave Hopfield-like algorithm that has been successfully tested in computer simulations of concrete pattern recognition applications [1, 2, 3].

The model proposed by Trugenberger makes a link between Artificial Neural Network (ANN)-like and logic-gate-based branches of quantum pattern recognition. The model is related to the fact that a special case of Hebbian memory-storage is equivalent to quantum-implementable NOT XOR gate [8]. However, some critical view had been added indicating that the advantage of quantum states as memories for classical information is not clearly demonstrated.

Ventura and Martinez have built a model of quantum associative memory where the stored patterns are considered to be basis states of the memory quantum state (*inclusive* method of quantum superposition). Unitary rotation operators increase the probability to recall the basis state associated with the input pattern [12, 9, 10]. The retrieval algorithm is based on the well known Grover's quantum search algorithm in an unsorted database, which is also an amplitude amplification of the desired state [6]. In order to overcome the limitation of that model to only solve the completion problem by doing data retrieving from noisy data, Ezhov *et al.* have used an *exclusive* method of quantum superposition and Grover's algorithm with distributed query [5]. However, their model still produces non-negligible probability of irrelevant classification.

Recently, Zhou and Ding have presented a new quantum multi-pattern recognition method, based on the improved Grover's algorithm proposed by Li and Li [7], which can recognize multi-pattern simultaneously with the probability of 100% [13]. The method introduces a new design scheme of initializing quantum state and quantum encoding on the pattern set. However, there is an important constraint that the rate of the number of the recognized pattern on the total patterns should be over $\frac{1}{3}$.

This paper suggests, through an *oracle* operator \mathcal{I}_M , two modifications of the query that can optimize data retrieval of correct multi-patterns simultaneously in the Ezhov's model of quantum associative memory without any constraint. In the first modification, \mathcal{I}_M invert only the probability amplitudes of the memory patterns states as in the Ventura's model. In the second modification, \mathcal{I}_M invert the probability amplitudes of all the states over the average amplitude of the query state centered on the m patterns of the learning.

The main steps of the approach of these models are [10]:

- Construction of a quantum classification system that approximates the function from which set of patterns M (*memory states*) was drawn;
- Classification of instances using appropriated Grover's quantum search algorithm (the time evolution step);
- Observation of the system.

The paper is organized as follows: Section 2 briefly present basic ideas of Ventura's and Ezhov's models respectively. Section 3 is used to introduce our

approach and show the comparing results with the above mentioned models. In Section 5 we summarize the paper.

2 Ventura's model and Ezhov's model

The main purpose of the quantum associative memory build by Ventura and Martinez is *pattern completion* [12]. That is, it can restore the full pattern from partial, but exact, part one. The memory use a storage algorithm and Grover's quantum search algorithm for retrieving the patterns.

Grover's quantum search algorithm can be considered as a rotation of the state vectors in two-dimensional Hilbert space generated by the initial and target vectors[6]. The amplitude of the target state increases towards its maximum while the amplitudes of other states decreases after a certain number of iterations.

Each neuron of the memory is a qubit that can be in a state $|0\rangle$, $|1\rangle$ or a superposed state $\alpha|0\rangle + \beta|1\rangle$, with $|\alpha|^2 + |\beta|^2 = 1$. $|\alpha|^2$ and $|\beta|^2$ are respectively the probability of state $|0\rangle$ and state $|1\rangle$. As with a register of n qubits, one can compute at the same time all the 2^n numbers by using the following superposition state

$$|\psi\rangle = \sum_{x=0}^{2^n-1} c_x |x\rangle, \quad \sum_{x=0}^{2^n-1} |c_x|^2 = 1, \quad (1)$$

a quantum associative memory can learn or store 2^n patterns. In the standard Grover's algorithm, Eq. (1) is obtained by applying n times the Walsh-Hadamard gate

$$W = \frac{1}{\sqrt{2}}(|0\rangle\langle k| + (-1)^k |1\rangle\langle k|), \quad k = \{0, 1\}, \quad (2)$$

to the initial state $|0\rangle$.

2.1 Storage algorithm

To generate a quantum register (1) in the superposition of only desired states from an initial state of n qubits of the network (*inclusive* method), Ventura and Martinez used the storage algorithm that they named algorithm of **initializing the amplitude distribution of a quantum state**. It works in a polynomial time and separate the initial state into the already stored patterns term and ready to process a new pattern term. The main operator of this algorithm is the 2-qubit controlled gate *state generation* [11,8],

$$\begin{aligned} CS^p &= |0\rangle\langle 0| \otimes \mathbb{I} + |1\rangle\langle 1| \otimes S^p = \text{diag}(\mathbb{I}, S^p), \\ S^p &= \begin{pmatrix} \sqrt{\frac{p-1}{p}} & -\frac{1}{\sqrt{p}} \\ \frac{1}{\sqrt{p}} & \sqrt{\frac{p-1}{p}} \end{pmatrix}, \end{aligned} \quad (3)$$

for $m \geq p \geq 1$, where $m \leq 2^n$ the number of pattern of length n to be store¹, each p is associate to a pattern. \mathbb{I} denotes the two-dimensional identity matrix. The operator (3) separates out the new pattern to be store by assigning to it small amplitude so that others operators can't act on it. To do that, we use three registers of n , $n - 1$ and 2-qubits:

- $|x\rangle = |x_1 \dots x_n\rangle$ the register where the m patterns of length n will be stored;
- $|g\rangle = |g_1 \dots g_{n-1}\rangle$ a register used like workspace to identify and mark a particular state;
- $|c\rangle = |c_1 c_2\rangle$ a register of two-qubits of control, that is the operator CS^p acts when $|c_1\rangle = |1\rangle$.

At the end of the algorithm there is no entanglement between the x -register and the two others which are respectively at $|0^{\otimes n-1}\rangle \equiv |0_1 \dots 0_{n-1}\rangle$ and $|0^{\otimes 2}\rangle \equiv |00\rangle$.

The simplified form of the storage algorithm is:

Algorithm 1 Simplified form of algorithm 4 (Ref. [11])

```

1:  $|\psi\rangle = |x_1 \dots x_n, g_1 \dots g_{n-1}, c_1 c_2\rangle \equiv |0_1 \dots 0_n, 0_1 \dots 0_{n-1}, 00\rangle$ ; {Initialize the register}
2: for  $m \geq p \geq 1$  do
3:    $|\psi\rangle = \text{FLIP}|\psi\rangle$ ; {Generate the state}
4:    $|\psi\rangle = \text{CS}^p|\psi\rangle$ ; {Apply  $\text{CS}^p$  operator}
5:    $|\psi\rangle = \text{SAVE}|\psi\rangle$ ; {Save the state}
6: end for
7:  $|\psi\rangle = \text{NOT}_{c_2}|\psi\rangle$ ;
8: Observe the system.
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The FLIP operator,

$$\text{FLIP} = \text{CNOT}_{(c_2 c_1)}^0 \text{CNOT}_{(c_2 x_j)}^0, (1 \leq j \leq n, z_{pj} \neq z_{(p+1)j}), \quad (4)$$

change the qubits state of the x -register when $|c_1\rangle = |0\rangle$ so that they correspond to the states $|P\rangle$ associated to patterns. The **SAVE** operator makes the state with the smaller amplitude a permanent representation of the pattern being processed and resets the other to generate a new state for the new pattern. At the end of the whole process the system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{m}} \sum_1^m |P\rangle. \quad (5)$$

called *blank memory* in the sense that all possible states have the same probability of being recovered upon measurement [8]. The number of steps of the storage algorithm is $\mathcal{O}(mn)$ which is optimal because reading each instance once cannot be done faster than that.

¹ Generally $m \ll 2^n$.

2.2 Retrieving algorithm

The associative memory proposed by Ventura and Martinez uses for retrieving information a modified version of **Grover's** search algorithm of an unsorted database. The original Grover's algorithm has been modified in order to include cases where not all possible pattern are represented and where more than one target state is to be found. The reader is referred to [12] for more details. It should be noted that Grover's algorithm use only $\mathcal{O}(\sqrt{\frac{N}{m}})$ steps to retrieve m elements in disordered list of $N = 2^n$ elements, while in classics algorithms the best use $\mathcal{O}(\frac{N}{m})$ steps.

2.3 Ezhov's model

As mentioned in the last section, the associative memory proposed by Ventura and Martinez can only do completion data. That is bits sequence shown to the network should be identical to a part of bits of one of memorized patterns. In order to overcome this limitation, Ezhov et *al.* have introduced a metric into the quantum search algorithm in the form of distributed queries [5]. The model is able to retrieve memory states with probability proportional to the amplitudes these states have in the query. Their quantum memory can retrieve valid stored patterns from a noisy data.

The model use the *exclusion learning approach* in which the system is in superposition of all the possible states, except the patterns states. If M is the set of patterns and m the number of patterns of length n ,

$$|\Psi\rangle = \frac{1}{\sqrt{N-m}} \sum_{x \notin M}^{N-1} |x\rangle, \quad N = 2^n. \quad (6)$$

In other words, the exclusion approach for the learning pattern included each point not in M with nonzero coefficient while those points in M have zero coefficients.

The distributed query is in the following superposed states

$$|Req^p\rangle = \sum_{x=0}^{N-1} Req_x^p |x\rangle, \quad (7)$$

where Req_x^p obey to binomial distribution

$$\|Req_x^p\|^2 = a^{d_H(p,x)} (1-a)^{n-d_H(p,x)}. \quad (8)$$

In equation (8)

- p marks the state $|p\rangle$ which is referred as the query center;
- $0 < a < \frac{1}{2}$ is an arbitrary value that regulates the width of the distribution;
- the **Hamming distance** $d_H(p, x) = |p - x|$ between $|x\rangle$ and $|p\rangle$ is an important tool which gives the correlation between input and output;

– the amplitudes are such that $\sum_x \|Req_x^p\|^2 = 1$.

The corresponding memory's algorithm is give by Algorithm 2 and the associate Brickman's diagram [4] by Figure 1.

Algorithm 2 Quantum associative memory with distributed queries (Ref. [5])

- 1: $|0_1 0_2 \dots 0_m\rangle \equiv |\bar{0}\rangle$; {Initialize the register}
 - 2: $|\Psi\rangle = A|\bar{0}\rangle = \frac{1}{\sqrt{N-m}} \sum_{x \notin M}^{N-1} |x\rangle$; {Learn the patterns using exclusion approach}
 - 3: **repeat**
 - 4: Apply the operator oracle \mathcal{O} to the register;
 - 5: Apply the operator diffusion \mathcal{D} to the register;
 - 6: $i = i + 1$;
 - 7: **until** $i > \Lambda$
 - 8: Observe the system.
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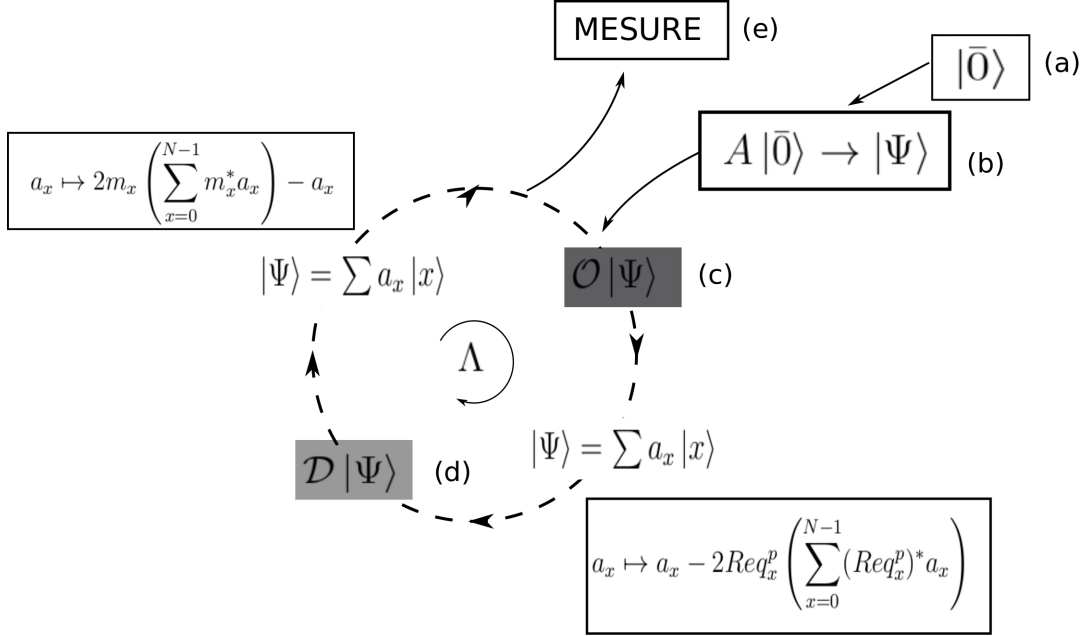


Fig. 1 The Brickman's diagram [4] of algorithm 2. (a) Qubits are initialize to the state $|\bar{0}\rangle$. (b) The operator A performs the learning of the set M using the *exclusion approach* and gives to all the possibles states the same probability amplitude a_x . (c) The operator oracle \mathcal{O} inverts probability amplitude of all the states over the average amplitude of the query state by changing a_x to $a_x - 2Req_x^p \left(\sum_{x=0}^{N-1} (Req_x^p)^* a_x \right)$. (d) The operator diffusion \mathcal{D} inverts the probability amplitude of the states of $|\Psi\rangle$ over their average amplitude and for the others over the value 0 by changing the amplitudes of probability a_x to $2m_x \left(\sum_{x=0}^{N-1} m_x^* a_x \right) - a_x$. (e) After Λ applications of steps (c) and (d) the system is observed.

In the Algorithm 2 or in the associate Brickman's diagram of Figure 1,

- \mathcal{O} is the operator oracle which invert the phase of the query state $|Req^p\rangle$,

$$\mathcal{O} = \mathbb{I} - (1 - e^{i\pi})|Req^p\rangle\langle Req^p|, \quad (9)$$

$$\mathcal{O} : a_x \mapsto a_x - 2Req_x^p \left(\sum_{x=0}^{2^n-1} (Req_x^p)^* a_x \right), \quad (10)$$

where a_x is the probability amplitude of the state $|x\rangle$.

- \mathcal{D} is the operator diffusion which invert the probability amplitude of the states of $|\Psi\rangle$ over their average amplitude and for the others over the value 0.

$$\mathcal{D} = (1 - e^{i\pi})|\Psi\rangle\langle\Psi| - \mathbb{I}, \quad (11)$$

$$\mathcal{D} : a_x \mapsto 2m_x \left(\sum_{x=0}^{N-1} m_x^* a_x \right) - a_x. \quad (12)$$

where m_x is the probability amplitude of a state of $|\Psi\rangle$.

- Λ is the number of iterations that whilst the maximal value of amplitudes, which must be as far as possible nearest to an integer,

$$\Lambda = T\left(\frac{1}{4} + \alpha\right), \quad T = \frac{2\pi}{\omega}, \quad \alpha \in \mathbb{N}, \quad (13)$$

with the Grover's frequency

$$\omega = 2 \arcsin B, \quad B = \frac{1}{\sqrt{N-m}} \sum_{x=0, x \notin M}^{N-1} Req_x^p. \quad (14)$$

Example 1 In order to help clarify, consider a 3-qubits memory where the patterns for the learning are $|010\rangle = |2\rangle$ and $|100\rangle = |4\rangle$ and the distributed query centered on $|011\rangle = |3\rangle$. For $a = \frac{1}{4}$,

$$|Req^3\rangle = \frac{\sqrt{3}}{8}|0\rangle + \frac{3}{8}|1\rangle + \frac{3}{8}|2\rangle + \frac{3\sqrt{3}}{8}|3\rangle + \frac{1}{8}|4\rangle + \frac{\sqrt{3}}{8}|5\rangle + \frac{\sqrt{3}}{8}|6\rangle + \frac{3}{8}|7\rangle, \quad (15)$$

$$B = \frac{1}{\sqrt{2^3-2}} \sum_{x=0, x \neq 2, x \neq 4}^7 Req_x^3 = \frac{6+6\sqrt{3}}{8\sqrt{6}}, \quad (16)$$

$$\omega = 2 \arcsin \frac{6+6\sqrt{3}}{8\sqrt{6}} = 0.63\pi \Leftrightarrow T = 3.17 \implies \text{for } \alpha = 1, \Lambda = 4. \quad (17)$$

The steps 4 and 5 of the Algorithm 2 will be repeated 4 times.

The operator oracle (10) is

$$\mathcal{O} = \mathbb{I} - \frac{1}{32} \left(\sqrt{3}|0\rangle + 3|1\rangle + 3|2\rangle + 3\sqrt{3}|3\rangle + |4\rangle + \sqrt{3}|5\rangle + \sqrt{3}|6\rangle + 3|7\rangle \right) \left(\sqrt{3}\langle 0| + 3\langle 1| + 3\langle 2| + 3\sqrt{3}\langle 3| + \langle 4| + \sqrt{3}\langle 5| + \sqrt{3}\langle 6| + 3\langle 7| \right). \quad (18)$$

As after the learning process the quantum memory is in the state

$$|\Psi\rangle = \frac{1}{\sqrt{6}}(|0\rangle + |1\rangle + |3\rangle + |5\rangle + |6\rangle + |7\rangle), \quad (19)$$

the operator diffusion (12) is

$$\mathcal{D} = \frac{1}{3}(|0\rangle + |1\rangle + |3\rangle + |5\rangle + |6\rangle + |7\rangle)(\langle 0| + \langle 1| + \langle 3| + \langle 5| + \langle 6| + \langle 7|) - \mathbb{I}. \quad (20)$$

At the end of the 4 iterations the register is in the state

$$|\Psi^4\rangle = -0.257|0\rangle + 0.031|1\rangle + 0.683|2\rangle + 0.531|3\rangle + 0.228|4\rangle - 0.257|5\rangle - 0.257|6\rangle + 0.031|7\rangle. \quad (21)$$

The probability to retrieve the memory states, $|2\rangle$ and $|4\rangle$, is $0.683^2 + 0.228^2 = 51.85\%$. As expected, the memory state closest in Hamming distance to the query center state, $|2\rangle$, presented the best probability (46.65%).

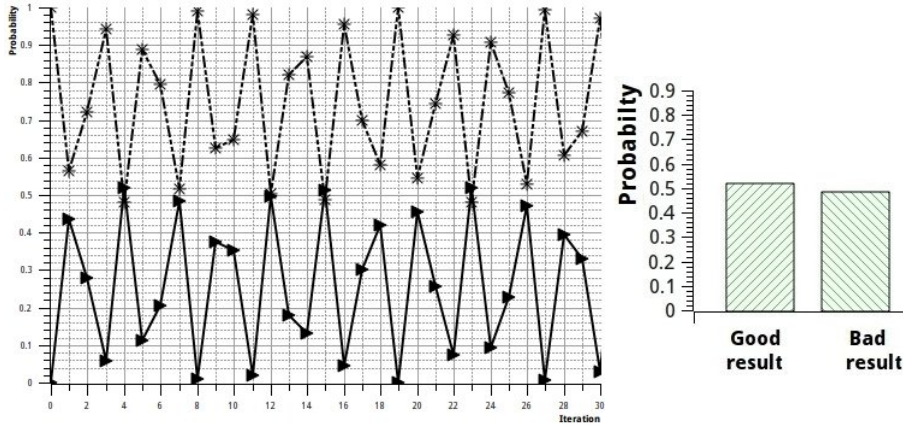


Fig. 2 Probability evolution with the number of iterations in Ezhov's model for 3-qubits memory patterns. The solid line represents the probability P_c of a correct recognition and the dotted line the probability P_w of an incorrect recognition.

Fig. 3 Probability of correct recognition for a set of two example patterns in Ezhov's model.

Figure 2 shows the probability of observing the correct recognition upon system measurement versus Grover's search iterations. The solid line represents the probability P_c of a correct recognition and the dotted line the probability P_w of an incorrect recognition. Note that the periodic nature of the algorithm clearly appears, and it can be seen that the probability of success P_c is maximized after four iterations. At $\Lambda = 4$, the ratio

$$\frac{P_c}{P_w} = 1.08, \quad (22)$$

that could be considered as the recognition efficiency of the memory patterns, shows that the confidence that the recognition is correct fair in the Ezhov's model. This clearly appears in the Figure 3 that gives the graphic representation of the probabilities of correct and bad recognition of this example.

It should be pointed out that if the approximation

$$B \simeq \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} Req_x^p, \quad N \gg m, \quad (23)$$

is wrongly use in Example 1, the number of iterations increase to $\Lambda = 9$ and the probability to retrieve the memory states $|2\rangle$ and $|4\rangle$ is reduce to 36.00% and therefore the recognition efficiency is

$$\frac{P_c}{P_w} = 0.56. \quad (24)$$

3 Improved quantum associative algorithm with distributed query

In order to improve the quantum associative memory with distributed query such that it optimize the probability of retrieving the learned patterns, even for the biggest Hamming distance from the query center, we proposed the Algorithm 3 illustrate by the Brickman's diagram of Figure 4, with an operator \mathcal{I}_M .

Algorithm 3 Improve quantum associative memory with distributed query

- 1: $|0_1 0_2 \dots 0_n\rangle \equiv |\bar{0}\rangle$; {Initialize the register}
 - 2: $|\Psi\rangle = A|\bar{0}\rangle = \frac{1}{\sqrt{N-m}} \sum_{x \notin M}^{N-1} |x\rangle$; {Learn the patterns using exclusion approach}
 - 3: Apply the operator oracle \mathcal{O} to the register;
 - 4: Apply the operator diffusion \mathcal{D} to the register;
 - 5: Apply operator \mathcal{I}_M to the register;
 - 6: Apply the operator diffusion \mathcal{D} to the register;
 - 7: **repeat**
 - 8: Apply the operator oracle \mathcal{O} to the register;
 - 9: Apply the operator diffusion \mathcal{D} to the register;
 - 10: $i = i + 1$;
 - 11: **until** $i > \Lambda - 2$
 - 12: Observe the system.
-

Two cases we be will considered for the operator \mathcal{I}_M :

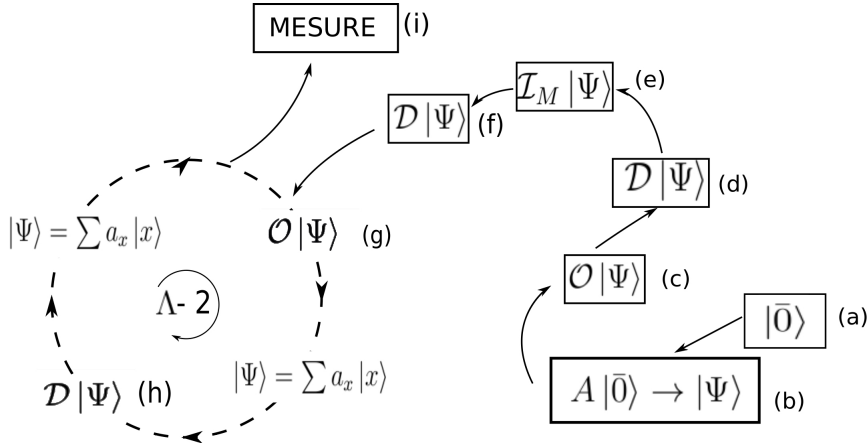


Fig. 4 Brickman's diagram [4] of Algorithm 3. The steps here are the same as in the diagram of Figure 1, with however two modifications: (e) The new operator \mathcal{I}_M acts as the operator oracle and (f) the operator diffusion \mathcal{D} is apply again before the $\Lambda - 2$ iterations.

C1: \mathcal{I}_M invert only the phase of the memory patterns states as in the Ventura's model,

$$\mathcal{I}_M = \mathbb{I} - (1 - e^{i\pi})|\varphi\rangle\langle\varphi|, \quad |\varphi\rangle\langle\varphi| = \sum_{x \in M} |x\rangle\langle x|, \quad (25)$$

$$\mathcal{I}_M : a_x \mapsto \begin{cases} -a_x & \text{if } |x\rangle \in M \\ a_x & \text{if not.} \end{cases} \quad (26)$$

$\forall x \in M$, the grover operator act as

$$\begin{aligned} \mathcal{D}\mathcal{I}_M|\varphi\rangle &= (2|\Psi\rangle\langle\Psi| - \mathbb{I} + 2|\varphi\rangle\langle\varphi|)|\varphi\rangle \\ &= 2|\Psi\rangle\langle\Psi|\varphi\rangle - |\varphi\rangle + 2|\varphi\rangle\langle\varphi|\varphi\rangle = |\varphi\rangle. \end{aligned} \quad (27)$$

C2: \mathcal{I}_M is formally identical to the operator oracle \mathcal{O} of Eq. (10),

$$\mathcal{I}_M : a_x \mapsto a_x - 2REQ_x \left(\sum_{x=0}^{N-1} (REQ_x)^* a_x \right), \quad (28)$$

with

$$\|REQ_x\|^2 = \frac{1}{k} \sum_p a_b^{d_H(b,x)} (1 - a_b)^{n-d_H(b,x)}, \quad (29)$$

where we consider that the distribution have k centers and $0 < a_b < \frac{1}{2}$ is an arbitrary value that regulates the width distribution around the center b . But in this paper, we will consider that

$$\|REQ_x\|^2 = \frac{1}{m} \sum_{b \in M} a'^{d_H(b,x)} (1 - a')^{n-d_H(b,x)}. \quad (30)$$

where m is the number of patterns for the learning, b is an item of the set M of patterns, and we choose the case where $a' = a_b$ is the same for all the patterns.

It is noteworthy that using a straight forward approach to classification and employing Grover's search, Ventura have found that the exclusion method exhibit the lowest overall probability of irrelevant classification compared to inclusion method [10]. This explains why the exclusion method is use in the Algorithm 3.

4 Simulations and results

Consider the data of Example 1, the evolution of probabilities with the number of iterations are plot in Figures 5 and 7 for **C1**-algorithm and **C1**-algorithm respectively.

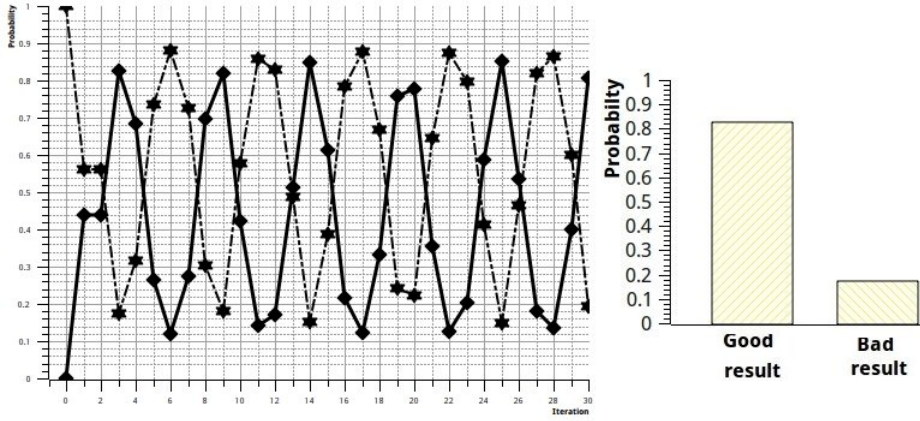


Fig. 5 Probability evolution with the number of iterations for **C1**-algorithm for 7-qubits memory patterns. The solid line represents the probability P_c of a correct recognition and the dotted line the probability P_w of an incorrect recognition.

Fig. 6 Probability of correct recognition for a set of two example patterns for **C1**-algorithm.

For **C1**-algorithm,

$$\mathcal{I}_M = \mathbb{I} - (|2\rangle\langle 2| + |4\rangle\langle 4|), \Lambda = 25 (T = 11). \quad (31)$$

Therefore, the steps 8 and 9 of Algorithm 3 are repeated 23 times. At the end of the algorithm, the register is in the state

$$\begin{aligned} |\psi^{25}\rangle = & -0.137|0\rangle + 0.0231|1\rangle - 0.876|2\rangle + 0.301|3\rangle - 0.292|4\rangle - 0.137|5\rangle \\ & - 0.137|6\rangle + 0.0231|7\rangle. \end{aligned} \quad (32)$$

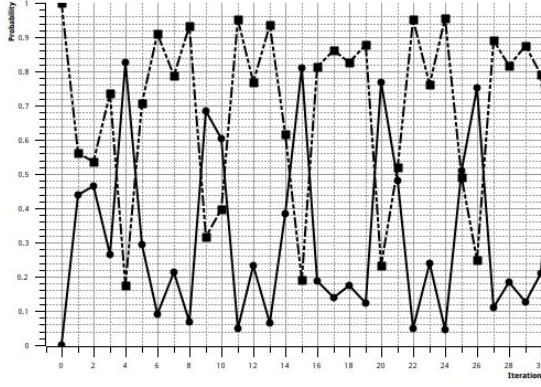


Fig. 7 Probability evolution with the number of iterations for **C2**-algorithm for 7-qubits memory patterns. The solid line represents the probability P_c of a correct recognition and the dotted line the probability P_w of an incorrect recognition.

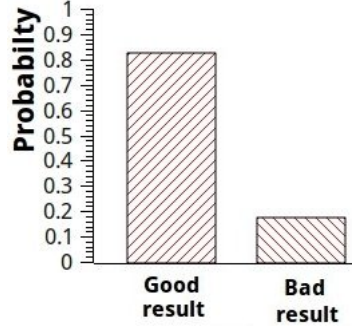


Fig. 8 Probability of correct recognition for a set of two example patterns for **C2**-algorithm.

The probability to retrieve the memory states, $|2\rangle$ and $|4\rangle$, is $0.86266^2 + 0.28755^2 = 85.23\%$. The ratio

$$\frac{P_c}{P_w} = \frac{0.852253}{0.1477} = 5.77, \quad (33)$$

shows how higher recognition efficiency of **C1**-algorithm compared to Ezhov's model.

For **C2**-algorithm, by choosing $\alpha' = 0.1$, one find

$$\begin{aligned} |REQ\rangle = & 0.285|0\rangle + 0.095|1\rangle + 0.607|2\rangle + 0.202|3\rangle + 0.607|4\rangle \\ & + 0.202|5\rangle + 0.285|6\rangle + 0.095|7\rangle, \end{aligned} \quad (34)$$

$$\begin{aligned} \mathcal{I}_M = & \mathbb{I} - (0.285|0\rangle + 0.095|1\rangle + 0.607|2\rangle + 0.202|3\rangle + 0.607|4\rangle + 0.202|5\rangle \\ & + 0.285|6\rangle + 0.095|7\rangle)(0.285\langle 0| + 0.095\langle 1| + 0.607\langle 2| + 0.202\langle 3| \\ & + 0.607\langle 4| + 0.202\langle 5| + 0.285\langle 6| + 0.095\langle 7|). \end{aligned} \quad (35)$$

and $\Lambda = 4$. Therefore, the steps 8 and 9 of Algorithm 3 are repeated 2 times. At the end of the algorithm, the register is in the state

$$\begin{aligned} |\psi^4\rangle = & -0.107|0\rangle - 0.024|1\rangle + 0.772|2\rangle + 0.358|3\rangle + 0.477|4\rangle + 0.152|5\rangle \\ & - 0.107|6\rangle - 0.024|7\rangle. \end{aligned} \quad (36)$$

The probability of correct recognition of memory states $|2\rangle$ and $|4\rangle$ is $0.772^2 + 0.477^2 = 82.69\%$, which is fairly lower than that of **C1**-algorithm.

The Table 1 which gives the summary of the relevant parameters of Ezhov's, **C1** and **C2** methods shows that despite the **C1**-algorithm is slower than **C2**-algorithm it leads to a better recognition efficiency of memory patterns. There is a significant gap between the recognition efficiency of **C1** and **C2** algorithms and that of Ezhov's model.

Table 1 Relevant parameters of Ezhov's, **C1** and **C2** methods for 3-qubits memory data of Example 1.

Method	Λ	P_c/P_w
Ezhov's	4	1.08
C1	25	5.77
C2	4	4.67

For a better comparison of **C1** and **C2** algorithms, consider a 7-qubits memory where the patterns for the learning are states $|23\rangle$, $|59\rangle$, $|61\rangle$, and $|110\rangle$ and the distribution query centered on $|60\rangle$. Figures 9 and 10 show the corresponding probabilities of observing the correct recognition upon system measurement versus the number of iterations of Grover's search for $a = 0.15$ and $a = 0.40$ respectively.

Table 2 Relevant parameters of Ezhov's, **C1** and **C2** methods for 7-qubits memory data.

Method	a	a'	Λ	P_c
Ezhov's	0.15	-	32	$< 10\%$
C1	0.15	-	20	57.04%
C2	0.15	0.10	10	51.45%
C2	0.15	0.40	13	18.35%
Ezhov's	0.40	-	21	$< 40\%$
C1	0.40	-	14	93.22%
C2	0.40	0.10	20	48.37%
C2	0.40	0.40	12	54.70%

It can be seen, as summarize in the Table 2, that

- the smaller the arbitrary value that regulates the width of the distribution a , for the three methods
 - the larger number of iterations Λ ;
 - the lower the recognition efficiency;
- for a giving value of a in the **C2**-algorithm, the closer a'
 - the lower number of iterations Λ ;
 - the larger the recognition efficiency;
- for any value value of $a \in [0, 1/2]$,
 - the best recognition efficiency of memory patterns is given by **C1**-algorithm and the poorest by the standard Ezhov's model;
 - **C2**-algorithm seem to be faster than **C1**-algorithm.

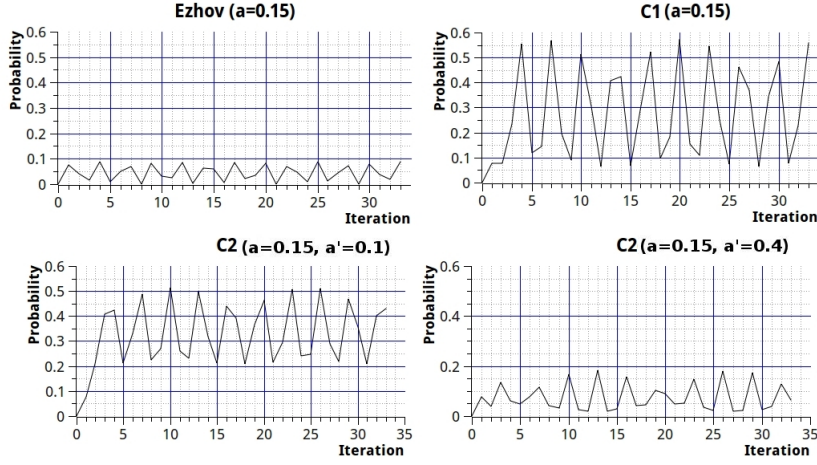


Fig. 9 Probability evolution with the number of iterations for 7-qubits memory patterns. The arbitrary value that regulates the width of the distribution is $a = 0.15$.

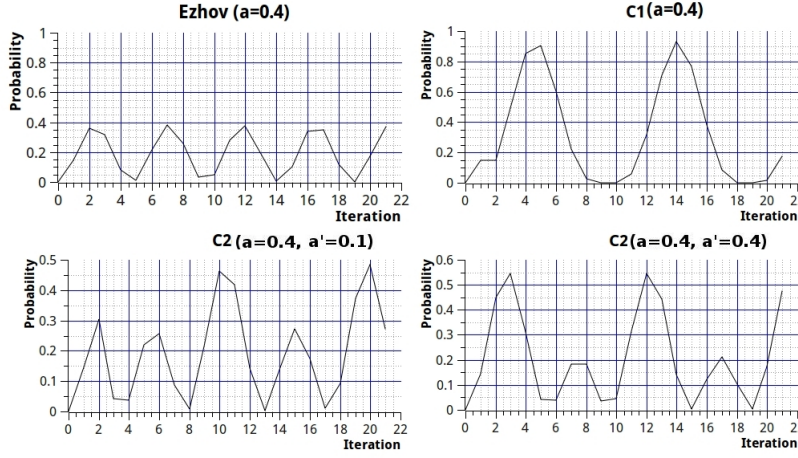


Fig. 10 Probability evolution with the number of iterations for 7-qubits memory patterns. The arbitrary value that regulates the width of the distribution is $a = 0.40$.

5 Conclusion

In this paper we have proposed a multi-pattern recognition method with a good rate of success based on an improved quantum associative algorithm with distributed query for any rate of the number of the recognition pattern on the total patterns. We have introduced an operator \mathcal{I}_M which acts as the oracle operator by considering two cases: the case **C1** where \mathcal{I}_M invert only the phase of the memory patterns states as in the Ventura's model; the case **C2**

where \mathcal{I}_M invert the probability amplitudes of all the states over the average amplitude of the query state centered on the m patterns of the learning. These improvements appeared as factors that increased constructive interferences so that the number of iterations is considerably reduced. Simulation results favour the **C1**-algorithm despite his high number of iterations compare to the **C2**-algorithm. Work is in progress to investigate other possibilities of improvement.

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A Learning algorithms

The algorithm 4 use the following gates:

- The hermitian 1-qubit NOT gate

$$\text{NOT} = |0\rangle\langle 1| + |1\rangle\langle 0| = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (37)$$

- Two forms of the hermitian 2-qubit controlled gate CNOT

$$\text{CNOT}^0 = \text{diag}(\text{NOT}, \mathbb{I}) \text{ and } \text{CNOT}^1 = \text{diag}(\mathbb{I}, \text{NOT}). \quad (38)$$

- Four forms of the hermitian 3-qubit Fredkin's gate F (a controlled-controlled NOT gate)

$$\text{F}^{00} = \text{diag}(\text{NOT}, \mathbb{I}, \mathbb{I}, \mathbb{I}), \text{F}^{01} = \text{diag}(\mathbb{I}, \text{NOT}, \mathbb{I}, \mathbb{I}), \quad (39)$$

$$\text{F}^{10} = \text{diag}(\mathbb{I}, \mathbb{I}, \text{NOT}, \mathbb{I}), \text{F}^{11} = \text{diag}(\mathbb{I}, \mathbb{I}, \mathbb{I}, \text{NOT}). \quad (40)$$

- And z_{pj} the values of different qubits of patterns, where we take $z_{(m+1)j} = 0^{\otimes m}$.

Algorithm 4 Learning the set M by initializing the amplitude distribution of a quantum state (Ref. [11])

```

1:  $|\psi\rangle = |x_1 \dots x_n, g_1 \dots g_{n-1}, c_1 c_2\rangle \equiv |0_1 \dots 0_n, 0_1 \dots 0_{n-1}, 00\rangle$ ; {Initialization}
2: for  $m \geq p \geq 1$  do
3:   for  $1 \leq j \leq n$  do
4:     if  $z_{pj} \neq z_{(p+1)j}$  then
5:        $|\psi\rangle = \text{CNOT}_{(c_2 x_j)}^0 |\psi\rangle$ ;
6:     end if
7:   end for
8:    $|\psi\rangle = \text{CNOT}_{(c_2 c_1)}^0 |\psi\rangle$ ;
9:    $|\psi\rangle = \mathbf{F}^p |\psi\rangle$ ;
10:   $|\psi\rangle = \mathbf{F}_{x_1 x_2 g_1}^{z_{p1} z_{p2}} |\psi\rangle$ ;
11:  for  $3 \leq k \leq n$  do
12:     $|\psi\rangle = \mathbf{F}_{x_k g_{k-2} g_{k-1}}^{\bar{z}_{pk1}} |\psi\rangle$ ;
13:  end for
14:   $|\psi\rangle = \text{CNOT}_{(g_{n-1} c_1)}^1 |\psi\rangle$ ;
15:  for  $n \geq k \geq 3$  do
16:     $|\psi\rangle = \mathbf{F}_{x_k g_{k-2} g_{k-1}}^{\bar{z}_{pk1}} |\psi\rangle$ ;
17:  end for
18:   $|\psi\rangle = \mathbf{F}_{x_1 x_2 g_1}^{z_{p1} z_{p2}} |\psi\rangle$ ;
19: end for
20:  $|\psi\rangle = \text{NOT}_{c_2} |\psi\rangle$ ;
21: Observe the system.

```
